The Effect of Professional Learning on Early Algebra Teachers' Content Knowledge in Nigeria

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Teachers' knowledge of the early algebra content that is to be taught is crucial for effective pedagogy and ensuring that the students' understanding of early algebra is not flawed. This article reports the findings of two of the activities that a group of in-service teachers participated in during a professional learning intervention program that was a part of a recent research study in Nigeria. The intervention program amongst other things focused on enriching the teachers' knowledge of some common students' misconceptions about the variable, expressions and equations. The teachers' algebra knowledge and PCK were enhanced as they examined some of the solutions they gave to two algebra word problems.

Teacher pedagogical content knowledge (PCK) and content knowledge are important for student success with the intended curriculum (Ball, Thames, & Phelps, 2008). In particular, algebra as the "gateway to higher mathematics" (Kaput, 1999, p. 134) needs to be effectively presented and taught for students to achieve success in it. Some teachers and textbooks use 'fruit salad algebra' in an attempt to promote student understanding of the concept of variable (Chick, 2009). The 'fruit salad' approach involves using the letter to represent a word or label for an object or item (Chick, 2009). Secondary school students have been found to commonly exhibit these misconceptions since they are just being introduced to the subject, and if not addressed, such misconceptions may impact students' understanding of higher algebra and other mathematics content (Ryan & Williams, 2007).

In particular, many students have misconceptions about variables and equations which are fundamental concepts for algebraic reasoning (Knuth, Alibali, McNeil, Weinberg, & Stephens, 2005). Studies show that students and pre-service mathematics teachers display misconceptions about the variable and equations, including using the algebraic letter as a word or object instead of as a quantity (Clement, 1982; MacGregor, 1991; Rosnick, 1981; Tanisli & Kose, 2013). However, there is limited research about in-service teachers' knowledge of the early algebra concepts which they teach to junior secondary students. Kieran (2007) called for more studies to examine the impact of professional learning that increases teachers' awareness of students' algebra misconceptions.

In Nigeria, students are introduced to the beginning concepts of variables, expressions, equations, functions and graphs in the first three years of secondary schooling. Nigerian secondary students' mathematics performances have remained poor and it appears that "the learned curriculum is at variance with the intended curriculum" (Festus, 2008, p. 148). This article reports the findings of two of the activities in which a group of in-service teachers participated during a professional learning intervention program that was a part of recent doctoral research work in Nigeria (Ladele, 2013). The aim was to identify errors (if

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any) due to misconceptions, before teachers were provided information about the common students' misconceptions in early algebra.

Method

As part of a larger study, 13 mathematics teachers from public and private secondary schools within an educational district in Lagos State, Nigeria participated in the professional learning program. They all taught the first year junior secondary school students (12 – 13 years) and some of them taught other junior classes as well. The teachers had either a three-year post-secondary Nigerian teaching qualification or were mathematics education degree holders. Only one of the teachers had more than five years of junior secondary mathematics teaching experience and 10 of them had between two and 10 years of overall mathematics teaching experience.

The first phase of the professional learning intervention program ran for two days during which the teachers updated their knowledge of students' likely thinking patterns that give rise to algebra misconceptions, and of language-based teaching approaches,. The two activities described in this paper occurred during this phase. At the onset of the program, the teachers completed the first activity. This involved providing individual solutions to two algebra questions, namely the student-professor and cheesecake problems, taken from Clement, Lochhead, and Monk (1981). Then the teachers jointly examined some students' responses to algebra word problems, and were provided information about the concepts of variable, expressions and equations. Later, the teachers in groups were given the opportunity to re-visit and provide reasons for some of the prior solutions they gave to the two questions. This forms the second activity reported in this paper.

The teachers were also introduced to the Newman strategy for understanding and solving word problems, a strategy which identifies the following five main stages with corresponding error types: reading, comprehension, transformation, processing skills and encoding (Newman, 1983a).

Instrument and Data Analysis

The two questions, particularly the student-professor problem, have been used in studies at various levels and with different ages (Clement, 1982; MacGregor, 1991; Rosnick, 1981). The two questions focused on the knowledge of the concept of variable and equality and are as follows.

1. Write an equation using the variables S and P to represent the following statement: "There are six times as many students as professors at this university." Use S for the number of students and P for the number of professors.
2. Write an equation using the variables C and S to represent the following statement: "At Mindy's restaurant, for every four people who ordered cheesecake, there are five people who ordered strudel. " Let C represent the number of cheesecakes and S represent the number of strudels ordered.

The words 'cheesecake' and 'strudel' were changed to 'cake' and 'sandwiches' respectively since the former were not common/familiar words within the Nigerian context.

Each teacher's written answers were collected and marked, and the identified responses were categorised and interpreted using MacGregor's (1991) classifications. The teachers' group responses to some of their own solutions were also collated and described.

Results and Discussion

Question 1 required the 13 teachers to use both the knowledge of the variable as a quantity and the knowledge of ratios. Only one teacher wrote the correct answer while half of the teachers used the variable to represent a word. The table below provides the response categories that emerged for the question, and the interpretation. The three tables that feature in this section are part of the doctoral thesis.

Table 1
Teachers' Responses to Question 1 (n=13)

Question	Responses	Number	Interpretation
Write an equation using the variables S and P to represent the following statement: "There are six times as many students as professors at this university." Use S for the number of students and P for the number of professors.	$S = 6p$	1	Correct answer
	$6s = p$	5	Reversal error
	$6ps$	3	Product-sum confusion
	$6s>p$	2	Inequation
	$6s + 6p$	1	Lack of equation
	$6s + p$	1	
	Representation: *s* represents students, *p* represents professor	6	Letter as a label

Of the 13 teachers, only one correctly answered the question. Six of them wrote and used the letters 's' and 'p' as representing student and professor respectively, while two of them wrote the answer as an expression. The reversal error seen in some responses revealed that some of the teachers may have had limited knowledge of some concepts. This error was also common in other studies (Clement, 1982; Lochhead & Mestre, 1999; Rosnick, 1981)

Question 2 demanded the same content knowledge but with the additional requirement of the ratio value being more than one. There were no correct responses given to this question. Unfortunately, all of the teachers provided incorrect responses to the question. Clements's students also had a lower percentage pass on this question. The question thus appeared to be more difficult than Question 1.

The majority of the teachers made use of the letter as a word/object, writing 'c' as 'cake' and 's' as 'sandwich' despite it being clearly written that it represented 'the number of ..'. This confirmed their earlier use in Question 1 of representing the algebra letter as a word. Like some of the in-service teachers in Chick's (2009) study, the teachers seemed to believe that the algebra letter may represent a word. Table 2 shows the teachers' responses and the interpretations.

Table 2
Teachers' Responses to Question 2 (n=13)

Question	Responses	Number	Interpretation
Write an equation using the variables *C* and *S* to represent the following statement: "At Mindy's restaurant, for every four people who ordered cake, there are five people who ordered sandwiches." Let *C* represent the number of cakes and *S* represent the number of sandwiches ordered.	$4s = 5c$	0	Correct answer
	$4c = 5s$	6	Reversal error
	$4c + 5s$	6	Lack of equation
	cs	1	Product/sum confusion
	Representation: *C* represents cake, *s* represents sandwich	4	Letter as a label

After the teachers collaboratively examined students' solutions to some algebra questions and updated their own knowledge of common student algebra misconceptions, they were introduced to the Newman strategy. The next day, the teachers examined in their wrong solutions to the two questions. Using the Newman (1983b) strategy, they identified and corrected their errors. Table 3 shows the responses to the two questions.

Table 3
Teachers' Diagnosis of Errors and Response to Questions 1 and 2 (n=13)

Question	Given responses	Diagnosis / Error	Likely misconception
Write an equation using the variables S and P to represent the following statement: "There are six times as many students as professors at this university." Use S for the number of students and P for the number of professors.	$S = 6p$	Correct answer	
	$6s = p$	Incorrect answer due to error in transformation	The use of the letter as a word, literal translation, using ratio as an equation
	$6ps$		The use of the letter as a word, use of expression as equation, left-right reading
	$6s+ p$		The use of the letter as a word, use of 'more' instead of 'times as many'
Write an equation using the variables C and S to represent the following statement: "At Mindy's restaurant, for every four people who ordered cake, there are five people who ordered sandwiches. Let C represent the number of cakes and S represent the number of sandwiches ordered.	4c = 5s	Incorrect answer due to transformation error	The use of the letter as a word, literal translation
	4c + 5s		The use of the letter as a word, using 'and' as literal visualisation, and writing an expression instead of an equation
		Correct response: 4s = 5c	

In both questions, the identified error was "transformation"; the teachers recognised that they had wrongly used the variable as a word instead of a quantity. Initially unaware of some misconceptions, the teachers were quick to identify their mistakes.

Early algebra is intended to serve as an effective introduction to the use of algebra as a mathematical tool. Thus it is essential that a solid foundation is laid. A teacher's subject content knowledge is important and impacts the intended curriculum content that is to be taught (Ball et al., 2008; Drageset, 2010).

Conclusion

Although the sample was small and may not be generalizable, the study revealed that 'fruit salad algebra' was still on the menu for some Nigerian teachers. The fact that the majority of the teachers involved had fewer than five years of junior secondary one teaching experience underscores the need for early in-service training. Studies have revealed that some pre-service teachers have also displayed algebra misconceptions, so it is not surprising that in-service teachers appeared to be unaware of some misconceptions about the concept of variable and equations. It also reiterates the important role that teacher knowledge of both content and students plays in ensuring that algebra concepts are properly understood by students and that the intentions of the early curriculum for mathematics are realised.

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